Problems: Week 1

1-1. You drop a ball from a height of y = 1m, on bouncing from the floor, if the collision is totally elastic, it will again rise to 1m and in principle this up and down excursion can last forever. Do you think this is a case of harmonic vibration? Justify your answer.

1-2. For the Earth-Mass system the potential energy of the mass M at a height h above the Earth is $P_g = Mgh$ ($h << R_E$) while for a spring of spring constant k the potential energy for change of length x is $P_{sp} = \frac{1}{2}kx^2$. So which is larger P_g for M = 0.1kg and h = 1m or P_{sp} for $k = 1.9 \times 10^4$ N/m and x = 0.01m ? Justify your answer. [R_E = radius of Earth, g = 9.8m/sec^2]

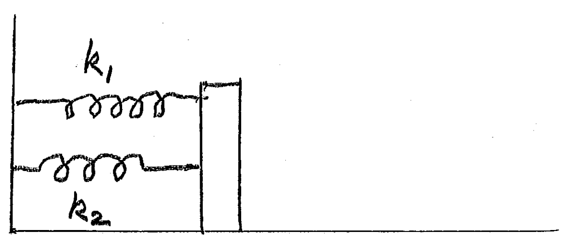
1-3. $P_g = Mgh$ is good only if $h << R_E$, otherwise $P_G = \frac{-GM_EM}{r}$, where M_E = mass of Earth $= 6 \times 10^{24}$kg, $G = 6.7\times10^{-11}\frac{N*m^2}{(kg)^2}$ and r is distance of M from center of Earth. How would you reconcile these two formulas?

1-4. The spring force is written as:

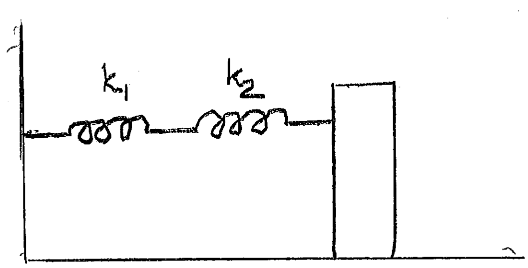
$$\underset{\longrightarrow}{F_{sp}} = -k\Delta x\hat{x}$$

Where k is the spring constant. Discuss why it is necessary to have the negative sign on the right in order to cause mechanical vibrations.

1-5. Given two springs of spring constants $k_1 = 100\text{N/m}$ and $k_2 = 50\text{N/m}$, what will be the effective spring constant if the springs are connected in parallel? Why?



1-6. What will be the effective spring constant if the springs of problem 1-5 are connected in series?



1-7. If your heart rate is 75 beats per minute what is the period and frequency of this "oscillation" ?

T = 0.8 sec

f = 1.25 Hz

1-8. For a spring-mass (m) oscillator the angular frequency $\omega = \sqrt{\frac{k}{m}}$ where k is the spring constant and the period is $T_0 = \frac{2\Pi}{\omega}$. For a fixed m by what factor would you change k to double (i) ω (ii) T_0? Why?

1-9. In problem 1-8 fix k and change m. Now what factors are needed to double (i) ω (ii) T_0? Why?

1-10. A spring-mass oscillator is represented by

$$x = 0.01\sin(12.56t + \frac{\Pi}{6})m$$

Where lengths are in meters and times in seconds. Calculate its (i) amplitude (ii) frequency (iii) phase (iv) maximum velocity (v) maximum acceleration

1-11. At room temperature in a solid, atoms oscillate typically at a frequency of 10^{13}Hz. Consider the case of copper (1mol is 0.064kg and has 6.02 x 10^{23}atoms). Suppose that while all the other atoms are at rest one atom has this frequency and estimate the effective spring constant of the "atomic spring" ?

1-12. An oscillator is represented by the equation

$$x = 0.05\sin(62.8t + \Theta)m$$

where t is in seconds.

Plot x as a function of t and calculate the times for the first zero and the first maximum if (i) $\Theta = \frac{\Pi}{3}$ (ii) $\Theta = \frac{-\Pi}{3}$?

1-13. Repeat problem 1-12 with

$$x = 0.05\cos(6.28t + \Theta)m$$

1-14. You inherit a grand father clock. Unfortunately, it loses 1min every hour. By what fraction must you change the length of the "pendulum" to make it run true? Why?